

Critical test of multi- j supersymmetries from magnetic moment measurements

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Abstract

Magnetic moment measurements in odd nuclei directly probe the distribution of fermion states and hence provide one of the most critical tests for multi- j supersymmetries in collective nuclei. Due to complexity of calculations and lack of data, such tests have not been performed in the past. Using the Mathematica software, we derive analytic expressions for magnetic moments in the $SO^{(BF)}(6) \times SU^{(F)}(2)$ limit of the $U(6/12)$ supersymmetry and compare the results with recent measurements in ^{195}Pt .

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The first examples of supersymmetry were found in the spectra of collective nuclei in the framework of the interacting boson-fermion model (IBFM) [1], and have attracted considerable attention during the past decade (see Refs. [1,2] for reviews). The single- j supersymmetries, which were first proposed [3,4], coupled a fermion in a single specific orbit to a collective boson core. As there are usually several single-particle orbitals available for the fermion, these models had limited success [2]. This restriction was overcome in multi- j supersymmetries [5] through the introduction of the pseudo-spin mechanism which allowed coupling of any number of single-particle orbits to the boson core. In a given supersymmetry scheme, the wave functions are fixed from the outset (i.e. independent of the Hamiltonian parameters) and require very specific couplings of the fermion states to the boson states. Since the wave functions are independent of the Hamiltonian, energies are not a sensitive test of the model, and one has to consider electromagnetic ($E2$ and $M1$) properties, and one- and two-nucleon transfer reactions. Of these, the $E2$ transition rates are not very sensitive to the single particle distributions because i) they are dominated by the boson contribution, the fermion contribution to the $E2$ matrix element (m.e.) being $1/N$ smaller than the boson part where N is the boson number, and ii) the fermion effective charges are all taken to be equal and do not distinguish between different orbitals. One-nucleon transfer reactions are sensitive to the single particle distributions, but the transfer operator contains up to two free parameters for each j -orbital which allows too much flexibility to provide a definitive test.

In contrast, the $M1$ properties are free of these shortcomings, namely, i) the boson and fermion contributions to the $M1$ m.e. are similar (in fact, the latter are usually larger), ii) the g -factors of all orbitals differ significantly, and iii) the $M1$ operator for odd nuclei does not contain any free parameters. Thus magnetic moments and $B(M1)$ values offer one of the most critical tests for probing the coupling schemes predicted by multi- j supersymmetries. Such tests, with one limited exception [6], have not been performed in the past due to the complexity of calculations and the lack of data. In this Letter, we point out that the algebraic computations that were previously deemed too complicated can be performed relatively easily using the Mathematica software [7]. We derive analytic expressions for magnetic moments in the $SO^{(BF)}(6) \times SU^{(F)}(2)$ limit of the $U(6/12)$ supersymmetry and compare the results with a new extensive set of data [8] in ^{195}Pt which, together with ^{194}Pt , furnishes one of the best known examples of this supersymmetry.

The wave functions in the $SO^{(BF)}(6) \times SU^{(F)}(2)$ dynamical symmetry can be expanded in terms of the boson-fermion product states as

$$\begin{aligned}
& |[N], [N_1, N_2], (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), \gamma, L, J\rangle \\
&= \sum_{\sigma_B, \tau_B, \gamma_B, L_B, j} \alpha(\sigma_B, \tau_B, \gamma_B, L_B, j; J) [|\sigma_B, \tau_B, \gamma_B, L_B\rangle \times |j\rangle]^{(J)}. \quad (1)
\end{aligned}$$

Here the quantum numbers N, σ, τ label the $U(6), O(6), O(5)$ groups respectively, and j denotes the $p_{1/2}, p_{3/2}, f_{5/2}$ single-particle orbits. The expansion coefficients α are given in terms of the isoscalar factors ξ for the group chain $U(6) \supset O(6) \supset O(5) \supset O(3)$ as [9,10]

$$\alpha(\sigma_B, \tau_B, \gamma_B, L_B, j; J) = (-)^{L_B+J+1/2} \hat{L} \hat{j} \left\{ \begin{matrix} L_B & L_F & L \\ 1/2 & J & j \end{matrix} \right\} \xi_{[N], (\sigma_B, 0, 0), (\tau_B, 0), L_B}^{[N], (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), L}, \quad (2)$$

where the curly bracket denotes a $6-j$ symbol and $\hat{j} = \sqrt{2j+1}$.

Introducing the boson and fermion creation (annihilation) operators $d_\mu^\dagger, a_{j\mu}^\dagger$ ($d_\mu, a_{j\mu}$), the $M1$ operator in the IBFM is given by

$$T(M1) = q_1 [d^\dagger \tilde{d}]^{(1)} + \sum_{jj'} t_{1jj'} [a_j^\dagger \tilde{a}_{j'}]^{(1)}, \quad (3)$$

where tilde denotes $\tilde{a}_{jm} = (-)^{j-m} a_{j,-m}$. The parameters q_1 and $t_{1jj'}$ in Eq. (3) are determined from the boson g-factor and single-particle matrix elements respectively. In the calculation of magnetic moments, in general, the cross terms with $j = l \pm 1/2$ can also contribute to the matrix elements. However, in the case of the $SO^{(BF)}(6) \times SU^{(F)}(2)$ dynamical symmetry, the $p_{1/2}$ and $p_{3/2}$ single particle states couple to different boson states, and hence such cross terms all vanish. This effectively simplifies the magnetic moment operator to diagonal terms only which we will rewrite in terms of the boson and single-particle angular momentum operators as

$$\begin{aligned} \hat{\mu} &= g_B \mathbf{L}_B + \sum_j g_j \mathbf{L}_j, \\ \mathbf{L}_B &= \sqrt{10} [d^\dagger \tilde{d}]^{(1)}, \\ \mathbf{L}_j &= -[j(j+1)(2j+1)/3]^{1/2} [a_j^\dagger \tilde{a}_j]^{(1)}. \end{aligned} \quad (4)$$

The g-factor for bosons, g_B , is determined from the supersymmetric even-even partner, and for (neutron) fermions from the single-particle (Schmidt) values as

$$g_j = \pm g_s / (2l + 1) \text{ for } j = l \pm 1/2, \quad (5)$$

where g_s is the spin g-factor of neutrons which, with the standard quenching factor of 0.6, has the value $g_s = 0.6g_s^{free} = -2.3$. The expectation value of the operator (4) in the states (1) can be calculated easily and gives for the magnetic moments

$$\mu_J = \sum_{\sigma_B, \tau_B, \gamma_B, L_B, j} (\alpha(\sigma_B, \tau_B, \gamma_B, L_B, j; J))^2 \frac{1}{2(J+1)} [(\bar{J} + \bar{L}_B - \bar{j})g_B + (\bar{J} - \bar{L}_B + \bar{j})g_j] \quad (6)$$

Here the bar denotes $\bar{J} = J(J+1)$. The advantages of magnetic moment measurements for testing the multi- j supersymmetries noted above, are evident from Eq. (6). For example, the magnetic moments are sensitive to the occupation of the fermion states because the g_j values differ significantly. If all the g-factors were identical (c.f. the usual assumption for E2 effective charges), i.e. $g_j = g_B = g$, then one would obtain $\mu_J = gJ$, which gives no information about the single-particle distribution.

Using the isoscalar factors given in Refs. [9,10], magnetic moments can be evaluated in a straightforward manner from Eq. (6). However, the algebraic manipulations required are very tedious and lengthy, and they have not been calculated in the past. We have overcome this problem by employing the Mathematica software [7] in the evaluation of Eq. (6) and obtained relatively simple expressions for the magnetic moments. As an example, we show an intermediate and the final step in the calculations for the ground-band states, $[[N], [N+1, 0], (N+1, 0, 0), (\tau, 0), 2\tau, 2\tau+1/2]$

$$\begin{aligned}\mu_{2\tau+1/2} = & [20(N+1)(N+2)(2\tau+5)]^{-1} \{ 40N(N+2)\tau(2\tau+5)g_B \\ & + 5(N+\tau+4)(N-\tau+1)(2\tau+5)g_{1/2} \\ & - 2(N-\tau)(N-\tau+1)(6\tau+5)g_{3/2} \\ & + (N^2(22\tau+35) + N(56\tau^2+202\tau+35) + 22\tau^3 + 213\tau^2 + 465\tau) g_{5/2} \}. \quad (7)\end{aligned}$$

Results at this step were checked for errors by ensuring that when one uses the same value g for all g-factors, $\mu_J = gJ$. Substituting g_j from Eq. (5) gives the final result

$$\mu_{2\tau+1/2} = \frac{2N\tau}{N+1}g_B - \frac{N^2(22\tau+35) + 15N(6\tau+7) + 8\tau^3 + 24\tau^2 + 108\tau + 70}{42(N+1)(N+2)(2\tau+5)}g_s, \quad (8)$$

which is not much more complicated than the corresponding expression for the quadrupole moment. The results of calculations for all low-lying levels of interest are listed in Tables I-III.

In the remainder, we compare the supersymmetry predictions with recent g-factor measurements in ^{195}Pt [8], one of the most studied and best examples of the $U(6/12)$ supersymmetry [5,9–15]. Table IV compares the experimental g-factors with the theoretical ones obtained from Tables I-III using $N = 6$, $g_B = 0.3$ (determined from ^{194}Pt [16]), and $g_s = -2.3$. In testing the quality of supersymmetry, the quantitative estimate

$$\phi = [\Sigma|exp - th|/\Sigma|exp|] \%,$$

has often been used when discussing energy levels and transition rates [2]. However, this is not very useful for the present comparisons of g-factors because it takes no account of either experimental uncertainties or the level of agreement that might reasonably be expected from a parameter-free calculation. We prefer, therefore, a graphical presentation followed by a case by case discussion.

We compare in Fig. 1 the experimental and theoretical g-factors for the stretched states $((\tau, 0), L = 2\tau)$ in the $(N+1, 0, 0)$ and $(N, 1, 0)$ representations, for which there are complete sets of data up to spin 9/2. Aside from the ground-state, to be discussed below, the agreement is good, especially for the $(N, 1, 0)$ representation. We emphasize that, in contrast to other tests, no free parameters are used in the present g-factor calculations. In view of this, the level of agreement is remarkable.

A significant deviation occurs for the ground state which is measured very accurately. Denoting the basis states by $|L_B \times j\rangle$, its decomposition is given by

$$|1/2_1\rangle = \sqrt{5/8} |0_1 \times 1/2\rangle + \sqrt{3/20} |2_1 \times 3/2\rangle - \sqrt{9/40} |2_1 \times 5/2\rangle. \quad (9)$$

The g-factors for the three basis states in Eq. (9) are, in order, 0.77, 1.37 and 0.37. The basis state $|2_1 \times 3/2\rangle$ which has the largest g-factor, has the smallest amplitude. Thus a possible explanation is that the ground state has a larger $p_{3/2}$ component than predicted by the supersymmetry. An alternative explanation may be the inadequacy of the simple $M1$ operator used for bosons. Such an operator cannot generate $M1$ transitions and therefore has been modified with the addition of the two-body operator, $[Q \times L]^{(1)}$. The quadrupole operator, Q , has a vanishing m.e. in the ground state, so this term could lead to very different contributions for the ground- and other states. The calculation of m.e. for two-body operators is, however, rather involved and this possibility needs to be pursued numerically.

For the remaining (non-stretched) states the data are too scant for graphical presentation. Nevertheless, we discuss the two $J = 5/2$ states at 455 and 544 keV, both of which have relatively large measured g-factors with large uncertainties. There are 13 possible basis states for $J = 5/2$ with $L_B = 2$ (twice), 3, 4(twice), and $j = 1/2, 3/2, 5/2$. The average g-factor for all these unmixed states is 0.30, the highest being 0.76 for the $|4 \times 3/2\rangle$ state. All the other basis states have g-factors around the average value or lower. It is extremely unlikely that both of these $5/2$ states are dominated by the $|4 \times 3/2\rangle$ configuration to the exclusion of many others. In other words, $g_{5/2} \approx 0.6$ for these states would be very difficult to explain in any particle-core model. The supersymmetry model is, nevertheless, consistent with the experimental results as the g-factors of these states may actually have values near the lower limits allowed by the experimental uncertainties.

In conclusion, we have presented a critical test of the $U(6/12)$ supersymmetry from g-factor measurements in $^{194-195}\text{Pt}$. The supersymmetry scheme makes definite predictions for the boson-fermion wave functions which have previously been tested through level energies, E2 transition rates and transfer reactions. In comparison with these observables, g-factors provide a more sensitive test of the wavefunctions because they depend directly on the single-particle distributions. Given that our g-factor calculations are parameter free, the level of agreement between theory and experiment is remarkable and, in general, supports the multi- j supersymmetry model in ^{195}Pt . Further g-factor measurements and similar tests are planned for other nuclei which evidence supersymmetry.

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FIGURES

FIG. 1. Comparison of g-factors for the stretched states $((\tau_1, \tau_2) = (\tau, 0), L = 2\tau)$ in the $(N + 1, 0, 0)$ and $(N, 1, 0)$ representations.

TABLES

TABLE I. Magnetic moments for the states $[[N], [N+1, 0], (N+1, 0, 0), (\tau, 0), L, J]$. The factor f denotes $f = N(N+2)$.

(τ_1, τ_2)	L	J	$(N+1)(N+2)\mu_J$
(0,0)	0	1/2	$-\frac{1}{6}(N+1)(N+2)g_s$
(1,0)	2	3/2	$\frac{9}{5}fg_B + \frac{1}{70}(11f + 23N + 70)g_s$
(1,0)	2	5/2	$2fg_B - \frac{1}{98}(19f + 27N + 70)g_s$
(2,0)	2	3/2	$\frac{9}{5}fg_B + \frac{1}{210}(17f + 5N - 38)g_s$
(2,0)	2	5/2	$2fg_B - \frac{1}{882}(139f + 115N + 134)g_s$
(2,0)	4	7/2	$\frac{35}{9}fg_B + \frac{1}{3402}(661f + 1321N + 5282)g_s$
(2,0)	4	9/2	$4fg_B - \frac{1}{378}(79f + 127N + 446)g_s$

TABLE II. Magnetic moments for the states $[[N], [N, 1], (N, 1, 0), (\tau, 0), L, J]$. The factor f denotes $f = N(N+4)$.

(τ_1, τ_2)	L	J	$(N+1)(N+3)\mu_J$
(1,0)	2	3/2	$\frac{9}{20}(f+11)g_B + \frac{3}{35}(5f+3)g_s$
(1,0)	2	5/2	$\frac{1}{2}(f+11)g_B - \frac{1}{294}(95f+141)g_s$
(2,0)	2	3/2	$\frac{9}{50}(7f+41)g_B + \frac{1}{210}(7f+141)g_s$
(2,0)	2	5/2	$\frac{1}{5}(7f+41)g_B - \frac{1}{882}(119f+597)g_s$
(2,0)	4	7/2	$\frac{7}{18}(7f+41)g_B + \frac{1}{3402}(1211f-2967)g_s$
(2,0)	4	9/2	$\frac{2}{5}(7f+41)g_B - \frac{1}{378}(119f-123)g_s$

TABLE III. Same as Table II but for the states with $(\tau_1, \tau_2) = (1, 1)$.

L	J	μ_J
1	1/2	$\frac{1}{3}g_B - \frac{1}{6}g_s$
1	3/2	$\frac{1}{2}g_B - \frac{1}{10}g_s$
3	5/2	$\frac{10}{7}g_B + \frac{59}{254}g_s$
3	7/2	$\frac{3}{2}g_B - \frac{3}{14}g_s$

TABLE IV. Comparison of supersymmetry predictions from Tables I-III with the experimental g-factors in ^{195}Pt [8].

(τ_1, τ_2)	L	J	$E_x(keV)$	$g_J = \mu_J/J$	
				experiment	theory
$(\sigma_1, \sigma_2, \sigma_3) = (7, 0, 0)$					
(0,0)	0	1/2	0	1.22 ^a	0.77
(1,0)	2	3/2	211	0.10(2) ^a	0.02
(1,0)	2	5/2	239	0.25(4)	0.40
(2,0)	2	3/2	525		0.20
(2,0)	2	5/2	544	0.60(20)	0.35
(2,0)	4	7/2	613	0.41(12)	0.13
(2,0)	4	9/2	667	0.34(4)	0.35
$(\sigma_1, \sigma_2, \sigma_3) = (6, 1, 0)$					
(1,0)	2	3/2	99	-0.41(4) ^a	-0.53
(1,0)	2	5/2	130	0.36(3) ^a	0.36
(2,0)	2	3/2	420		0.20
(2,0)	2	5/2	455	0.63(22)	0.30
(2,0)	4	7/2	508	0.16(2)	0.03
(2,0)	4	9/2	563	0.34(3)	0.35
(1,1)	1	1/2	222		0.97
(1,1)	1	3/2	199		0.25
(1,1)	3	5/2	389	0.16(4)	-0.04
(1,1)	3	7/2	450		0.27

^aFrom Ref. [17].